By-Davis, Robert B.
Mathematics Chapter.
Syracuse Univ., N.Y.; Webster Coll., Websters Grove, Mo.
Spons Agency-Office of Education (DHEW), Washington, D.C. Bureau of Research.
Bureau No-BR-5-1172
Note-21p.
EDRS Price MF-\$0.25 HC-\$1.15

Descriptors Algebra, *Curriculum, *Curriculum Development, Educational Objectives, *Elementary School Mathematics, Geometry, Instruction, Learning, *Mathematics, *Secondary School Mathematics, Teaching Methods

Suggestions are offered for more effective implementation and creativity in developing new mathematics programs. The new mathematics materials and projects have not significantly altered the educational experiences of students because of (1) difficulty in identifying distinctive features of the various projects, (2) widespread assumptions that the "new" projects are mainly more efficient routes to "the same old goals," (3) curriculum workers' regard of "new mathematics" projects as irrelevant to local school needs because of the assumption that all "new mathematics" projects must necessarily be content centered, (4) the lack of an extensive program of teacher education, and (5) difficulty in hiring competent new teachers. All the curriculum projects make some assumptions about the nature of schools relating to such ideas as (1) the increasing role of intrinsic motivation, (2) the emphasis on learning how to learn and on individualizing instruction, (3) the emphasis on creativity and divergent thinking on the open-ended aspect of learning, and (4) a growing concern over inadequacies in testing programs. The article concludes with a list of references concerning (1) planning for the future, (2) surveys and over-views, and (3) United States and International projects in mathematics. (RP)



3

ED028919

U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE OFFICE OF EDUCATION

VAI-D-233

:.- 67

THIS OOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATEO OO NOT NECESSARILY REPRESENT OFFICIAL OFFICE OF EDUCATION POSITION OR POLICY.

Mathematics Chapter

Robert B. Davis

Professor of Mathematics and Professor of Education

Syracuse University and Webster College

In the case of mathematics, "new curriculum" projects are not entirely new -- some of them have now been active for as long as 10 years. When we look at the panorama of mathematics projects, what do we see? If we look carefully, we can discern an array of materials, ideas, teaching procedures, films, and so on, which can be used creatively at the local level. Indeed, these various new materials can be used to accomplish many local objectives that go well beyond the narrowest idea of "curriculum."

Suppose, for example, that a local supervisor wishes to give his teachers professional experience that will take them beyond their usual classroom activities, but will not take them out of the classroom altogether. Such experiences are valuable because, among other things, they promote the growth of the individual teachers. For such a purpose, the new curricula can help: a teacher who acquires special competence in "new mathematics curricula" has something valuable to share with her colleagues. She can conduct inservice meetings, or can teach demonstration classes, and be well received by her colleagues because she gives them something of value.



SE004 12

¹Cf. Sheldon Gold, The Reaction of Teachers to the Methods of Teaching Modern Mathematics Using Madison Project Materials, School of Education, The City College of The City University of New York. (January, 1965).

Suppose, as a second example, a local supervisor wants to increase the diversity of experiences which fill a child's day in school. Again -- with proper selection -- the new curricula can help. Various projects have given attention to the kind of activity which the child encounters, and have devised activities of a varied and compelling nature: velocity and acceleration studies on automobiles can be performed in the school parking lot, and the data subsequently studied by means of graphs. Children can determine the height of the flagpole in the school yard, by means of similar triangles. "Secret" numbers may be written on a concealed piece of paper, and clues can be given, from which the children deduce what the numbers must be. Team games have been devised to give children practice in plotting points on Cartesian coordinates, working with signed numbers, and so on. Vectors and statistics enter into some experiments that can be performed on the breaking strength of various materials. Functions obtained from a wide variety of experiments can become the subjects of mathematical discussion.

Students can make up their own algebra, make up their own geometry, and even make up their own logic -- and then compare their different results. Probability and statistics can be learned through, among other things, studies of local automobile traffic, or of waiting times at a crowded restaurant, or of queing problems at service counters, and so on. Students can make use of digital computers to work on problems they program themselves. In place of a sedentary and passive experience, mathematics can become active, original, and exciting. What child does not thrill to rockets? Mathematics is an essential part of space operations, and we do not need to throw away all the excitement and keep only that which is dull and routine. (Nor, on the other hand, do we need to lose sight of essentials, and focus only on a superficial sophistication.)



The View on the Local Front:

1. Have the New Mathematics Projects Changed our Schools? The answer, of course, is that, for the most part, they have not. (In a very few schools they actually have, and the change has gone well beyond the "mathematics" class.)

Indeed, the actual change in the child's educational experience has generally been extremely slight. Why has the effect not been deeper? Some of the reasons appear to be these:

- i) School administrators face a difficult task in identifying the distinctive features of the various projects. The projects are different, and the differences are often important. This task is made even harder by the nearly universal tendency to speak of "the new math" as a single entity, which it surely is not.
- ii) There is a widespread, but erroneous presumption that the "new" projects are mainly more efficient routes to "the same old goals." In actual fact, most of the "new" projects differ from the traditional curriculum primarily by having new and different goals.
- iii) It appears that those school curriculum workers whose educational goals are heavily child-centered often regard "new mathematics" projects as irrelevant to local school needs because of the presumption that all "new mathematics" projects must necessarily be "content centered." Some, indeed, might properly be classified as "content centered." However, some of the "new mathematics" programs are more consistently child-centered than most traditional programs are.



iv) In order to make any seriously effective use of the new mathematics project materials, an extensive program of teacher education is usually essential -- especially at the elementary school or junior high school level, and sometimes even at the senior high school level. In a valuable article on "new curricula," Alexander Frazier goes further, and argues that in-service education is needed for "everybody concerned, including ourselves as leaders. ...we ourselves need to understand what is happening to us both locally and nationally."

Obviously, arrangements for serious study by many teachers and also by school curriculum leaders are not always easily made; the need for such study constitutes one more barrier to the effective local utilization of the "new curricutum" materials.

- v) Articulation, especially between elementary school, junior high school, and senior high school has always been a problem. Given the present rapid rate of change, plus the inevitable and basically valuable diversity of "new mathematics" materials, articulation is usually an acute problem today.
- vi) In hiring new teachers, it is next to impossible for a school to hire teachers who are well-educated in relation to modern mathematics, especially in a form that is consonant with the specific local program that the school has been developing. Probably the difficulty is so great that most schools do not even try.



¹Alexander Frazier,"Making use of national curriculum studies," Educational Leadership, Vol. 22, No. 2, November, 1964, pp. 101-131.

vii) As mathematics comes more fully into contact with science — which is now beginning to occur — a whole host of additional problems appear. Articulation difficulties are aggravated; mathematics teachers who do not like the manipulation of physical apparatus find themselves confronted with a need for it; scheduling problems arise in blocking mathematics and science periods together; new forms of faculty cooperation are required; and so on.

viii) Finally, the further progress of the "new mathematics curricula" is seriously impeded by the mistaken notion that the job has already been completed. As
many leading school superintendents and curriculum workers have recently pointed
out, the job has hardly been started. Extremely little change is usually observable
in local school programs — beyond, perhaps, a "new textbook adoption" which
hardly realizes the tremendous potential for a re-vitalized school program that is
contained within the new curriculum materials.

It is very easy for a school system to give the appearance of having "adopted the new mathematics," but it is very hard to achieve the reality of building a vigorous and cohesive local program that takes full advantage of the "new curriculum project" materials, ideas, and services. Many schools have done the former, but few schools have achieved the latter — and those few schools that have usually find themselves transformed into new schools as a result of their efforts.

2. The Projects: Trends, Emphases, and New Approaches. The mathematics projects were reported rather fully by Henderson in 1963. Some additional surveys are listed



¹Kenneth B. Henderson, "Mathematics," Chapter Six in <u>Using Current</u> Curriculum Developments, ASCD, 1963, pp. 49-58.

at the end of this chapter, and are worth consulting.

A unique new development in national curriculum work is the Cambridge Conference on School Mathematics. This group, associated particularly with M.I.T. and Harvard, but with wide membership, has set themselves the unique task of trying to plan for the next thirty years of evolution in the mathematics curriculum. This task entails evident hazards and limitations, but it is not foolish. He who does not plan for the future, and strive to build for the future, must accept the future. Progress is one of the products of our schools, quite as much as of our industries, and serious effort and energy must somehow be stolen from the immediate tasks of today and allocated to building for the years ahead. The cost to the present is not negligable, but only the short-sighted would refuse to pay the price.

Whether schools and colleges will generally join in a program of planning and working for rational evolution remains to be seen; but if the educator of 1965 shares responsibility for the school of 1995, can be hold back? We must account carefully for the future liabilities we incur when we commit all of our resources to the urgent needs of today and of the quite immediate future, with nothing allocated to working for the long-term future.

Obviously, the first report issued by the Cambridge Conference (entitled Goals for School Mathematics) is not a blueprint. It will surely be wrong in many of its suggestions, as will be revealed by explorations of the next few years. Nonetheless, such serious long-term planning is literally unprecendented in mathematics, and so the importance of this effort is very great indeed. The tone of the first report is optimistic, and assigns serious mathematical subjects — including calculus, a large amount of "advanced" algebra, probability, and analytic geometry — to the pre-college years of a student's life. Small



Of. Robert B. Davis, "The Evolution of School Mathematics," <u>Journal</u> of Research in Science Teaching Vol. 1 (1963), pp. 260-264.

exploratory efforts have already convinced many that roughly this level and amount of mathematics are well within the achievement of many students, but they will make severe demands upon teacher education programs in our colleges. They must also influence the qualifications we seek in all new teachers whom we hire, for many of those whom we hire today will be teaching in our schools in 1995.

The question must arise to anyone reading the <u>Goals</u> report that this is a considerable investment in mathematical study: will it be possible to include it within an over-all curriculum that fully recognizes the student's need to learn about himself, his society, and the world at large? While most educators — the present author definitely among them — would assign a greater priority to learning about oneself, one's society, and the world in general, with a lesser priority for the specific subject of mathematics, the answer to the question is nonetheless not clear. Not, that is, when one tries to think of 1994. Much mathematics taught in our schools today moves at an incredibly slow pace, and over—elaborates upon the obvious if not upon the trivial. Nor can we claim that the graduates of our schools have really learned about themselves, or about their society — few have.

If we imagine heroic efforts at strengthening our curriculum in all of its component parts, what will be the result? "Strengthening," of course, does not mean merely "adding to" — it means developing more suitable learning experiences in every area, and selecting wisely among them.

Beyond these remarks, we leave the report of the Cambridge Conference to speak for itself. The Conference is still actively at work, and presumably more will be heard from them in the future.



On a more immediate level, the following trends can be noted in new curriculum materials presently available:

- i) increasing efforts to combine mathematics and science, or at least to relate the two;
- ii) a more prominent role for <u>mathematical logic</u> in pre-college mathematics, beginning even as early as grades 5 and 6;
- iii) an ever-increasing tendency to <u>unify</u> "mathematics," and to <u>cast aside any</u>

 <u>division</u> into component parts, such as "arithmetic," "synthetic geometry," "analytic

 geometry," "algebra," "physical application," and so on;
- iv) an over-increasing importance, at the pre-college level, for calculus, analytic geometry, and statistics;
- v) a tentative introduction of <u>matrix algebra</u> at various grade levels, ranging from grade 5 through grade 12;
- vi) as the preceding five items suggest, a tendency to expect more from students, and particularly from bright students, at virtually all grade levels;
- vii) a greater diversity of kinds of learning experiences, especially the inclusion of physical experiments as part of the study of mathematics (cf. various examples cited earlier in this chapter);
- viii) within the past year or so, expecially, the "new curriculum" work (which had previously been almost entirely a suburban phenomenon) has been moving into



the heart of some of our largest cities, and eliciting a gratifying response from "culturally deprived" students.

Several additional matters deserve special comment: for one to g, the fact that the ability to discover patterns in abstract materials is a central and major component of mathematical thought has been receiving greater and greater emphasis in the last few years.

The fact that some new curriculum materials are largely content-centered, while others are largely child-centered, has been mentioned earlier. This difference should be of major concern in building local programs. (Actually, this may be an unfortunate dichotomy; perhaps it is better to think in other terms. It is, however, important to notice that "new mathematics" programs are of many different types; they mean different things when they speak of "discovery," or of "structure," and so on, and they make quite different uses of student experience, readiness-building, program step-sizes, and so forth.



To get the flavor of "discovery" teaching in today's classrooms, you may want to read especially: Henry D. Snyder, "An Impromptu Discovery Lesson in Algebra," The Mathematics Teacher, Vol. LVII, No. 6 (October, 1964), pp. 415-416.

David Clarkson, "Taxicab geometry, rabbits, and Pascal's triangle - discoveries in a sixth-grade classroom," The Arithmetic Teacher, Vol. IX, No. 6 (October 1962), pp. 302-313.

Donald Cohen, "A Lesson on Absolute Value," The Arithmetic Teacher, December, 1964, Vol. 11. No. 3, pp. 561 and 562.

Morris Pincus, "An Adventure in Discovery," The Arithmetic Teacher, Vol. 11, No. 1 (January, 1964, pp. 28-29.)

Sheldon Gold, "Graphing Linear Equations - A Discovery Lesson,"

The Arithmetic Teacher (to appear).

Some projects prepare entire programs, which may be adopted for a long sequence of grades -- perhaps even K-12 -- whereas other projects produce "components" or "pieces" that may be incorporated into local programs.

of special importance in mathematics curriculum work today is the widespread use of films showing actual classroom lessons. The best of the "new mathematics" work is so novel that it must be seen to be believed — or to be fully understood. No adequate approach to the new mathematics curricula can omit the use of these films. Films, with various philosophies and goals, are now available from: Educational Services, Incorporated (David Page); The Madison Project; The National Council of Teachers of Mathematics; The Mathematical Association of America; The School Mathematics Study Group; MINNEMAST; The UICSM (Max Beberman); The National Film Board of Canada (Caleb Gattegno); and other projects and organizations.

Mathematics is more international than many subjects, and this fact manifests itself in highly similar programs developing in the United States, in England, on the Continent, in Russia, in Africa, and elsewhere. (A few foreign or international projects and activities are listed at the end of this chapter.)²

An interesting recent development has been the notion of occasionally replacing "teaching" by something rather different, namely, putting the learner in a "responsive



¹Cf. Alexander Frazier, op. cit., p. 105.

²Cf. Howard F. Fehr, "Reform of Mathematics Education Around the World," The Mathematics Teacher, Vol. LVIII, No. 1, January 1965, pp. 37-44.

environment" in the sense of O.K. Moore and Richard Suchman. In this approach, the teacher himself may constitute the "responsive environment." In such a case, the student explores in whatever direction he chooses, and the teacher tries to answer questions, or to partially answer questions. The responsibility for devising questions and for shaping the inquiry lies with the student.

Finally, all new curriculum projects must, of course, make assumptions about the nature of schools in general. A recent trend has appeared, among some but not all projects, to assume a background of a new kind of school. This "new kind of school" would, in general, relate to such ideas as these:

The increasing role of intrinsic motivation; the emphasis on learning how to learn; the emphasis on individualizing instruction; the emphasis on creativity and divergent thinking; the increasing use of digital computers in relation to schools; the scheduling of classes to allow greater flexibility, and especially longer periods, when necessary, for laboratory experimentation; the present interest in physical materials such as Cuisenaire rods, Dienes blocks, and single-concept film loops;



Cf. Jerome D. Kaplan, "An Example of Student-Generated Sequences in Mathematics Instruction," The Mathematics Teacher, Vol. LVII, No. 5 (May, 1964) pp. 298-302.

For information on Suchman's important "inquiry" studies, write:

J. Richard Suchman, Inquiry Training Project, School of Education, University of
Illinois, Urbana, Illinois.

²Cf. Ronald Gross, "Two-Year-Olds Are Very Smart," <u>The New York Times</u>, September 6, 1964, Magazine Section, pp. 10-11.

Jerome S. Bruner, <u>The Process of Education</u>. Harvard University Press, 1963.

assignments" outside of school, outside lecturers, television, and other community

resources; a tendency to "ungrade" the entire school program; a greater emphasis on the open-ended aspect of learning; and a growing concern over inadequacies in our testing programs. It manifests itself in the concern for the structure of mathematics rather than for facts; and in the matter of teaching the subject itself, rather than some "simplified" version of the subject which lacks authenticity. It is involved, also, in the growing concern over indoctrination.

By far the most revolutionary prospect is conjured up by the important work using computer-based teaching machines, now being carried on by Patrick Suppes at Stanford University, and by Wax Beberman at the University of Illinois. The essential idea is that computer technology has advanced to the point where it is possible to install a computer in a school, and to let the computer guide the work of <u>each individual student</u>.

The student can "talk" to the machine by a variety of methods: by typing on an electric typewriter connected to the machine, by pointing or drawing with a "ray-of-light" pencil, or by pushing buttons. The computer can "talk" to the student by typing out its response on an electric typewriter; by displaying diagrams, pictures, (etc.) on a TV-tube; or by actual voice from pre-recorded tapes (as in tape-recorded telephone announcements, etc.) At present the only important mode of communication between student and machine that is lacking is the ability of the machine to respond to spoken language of the student, and considerable effort is under way to develop even this.

It seems reasonable to expect big things from the use of computer-based teaching machines. Indeed, this is probably the first piece of technology that has the potential of really transforming our schools into something quite different -- on the scale, for



example, that automobiles have transformed our cities, suburbs, and mode of life. One can imagine a school organized into many parts: one part of the school day would involve computer-based totally individualized instruction; another, small group seminar discussions; another, laboratory experimentation; another, lectures or television presentations; another, individual library work or independent reading. The potential is staggering, but this is not sheer fantasy. Professor Suppos' work, alone, is presently subsidized to the extent of \$2,000,000, provided by men and institutions of sound judgement and realistic vision.

The Suppes group have, at present, an entire course in mathematical logic available for machine instruction, and are now developing an entire course in geometry. The Beberman group at Illinois have additional course-work already in existence. Trials are now taking place in California and Illinois, and may soon begin in New York City.

An additional aspect of the "new mathematics" efforts is provided by the recent work at the nursery school and kindergarten level, being undertaken by various groups (and perhaps especially by Joy C. Levy of Princeton, New Jersey).

The ideas and materials that are coming from the national curriculum projects in mathematics range from the brilliant to the foolish, from the wise to the mistaken. By themselves they cannot, however, possibly be effective. That will depend upon what is done with them at the local level. There is a creative, courageous, far-sighted role -- not without hazards on every side -- that is reserved exclusively for teachers and curriculum workers at the local level. The future depends upon them.



References

Planning for the Future

Goals for School Mathematics, the Report of the Cambridge Conference on School Mathematics, Houghton Mifflin Co., Boston, Massachusetts, 1963. (N.S.F.)

Surveys and Over-views

- Z. P. Dienes, The United Nations Educational, Scientific, and Cultural Organization Curriculum Study (UNESCO) (elementary grades), University of Adelaide, Adelaide, Australia. (UNESCO)
- Howard F. Fehr, "Reform of Mathematics Education Around the World," The Mathematics Teacher, Vol. LVIII, No. 1, January 1965, pp. 37-44.
- E. E. Moise, "The New Mathematics Programs," Chapter Nine in Modern Viewpoints in the Curriculum, edited by Paul C. Rosenbloom, McGraw-Hill, 1964.
- National Science Foundation, Science Course Improvement Projects, July, 1964 (Superintendent of Documents, U. S. Government Printing Office, Washington, D.C., 20402).
- Organization for European Economic Comperation, Synopses for Modern Secondary School Mathematics (available from O.E.E.C., Chateau de la Muette, Paris, France).
- Robert W. Ritchie, John G. Kemeny, and Robin Robinson, New Directions in Mathematics, Prentice-Hall, Inc., 1963.
- Adrian B. Sanford, International Study Group for Mathematics Learning (all grades), 200 California Ave., Palo Alto, California.
- T. Varga, and W. Servais, <u>Unesco Curriculum Study</u> (all grades), Muzeum Körut 6-8, Budapest, Hungary (UNESCO).



United States Projects

- Max Beberman, University of Illinois Committee on School Mathematics (UICSM) (Secondary, Elementary), University High School, College of Education, University of Illinois, Urbana, Illinois, 61803. (Carnegie Corporation and N.S.F.)
- E. G. Begle, School Mathematics Study Group (SMSG) (Elementary, Secondary), School of Education, Stanford University, Stanford, California 94305. (N.S.F.)
- Father Stanley J. Bezuska S.J., <u>Boston College Mathematics Institute's Contemporary Mathematics Program</u> (Secondary), Department of Mathematics, Boston College, Chestnut Hill 67, Massachusetts. (N.S.F.)
- Herman Branson, <u>Pre-College Center Program</u> (grade 12), Physics Department, Howard University, Washington D. C., 20001. (Carnegie Corporation)
- George Cunningham, et al., <u>The Greater Cleveland Mathematics Program</u> (Elementary and Secondary), Educational Research Council of Greater Cleveland, <u>75 Public Square</u>, Cleveland 13, Ohio.
- Robert B. Davis, The Madison Project (all grades), Webster College, Webster Groves, Missouri, 63119. (N.S.F. and U.S. Office of Education)
- E. Glenadine Gibb, Foundations of Mathematics for Elementary School Teachers, Department of Mathematics, State College of Iowa, Cedar Falls, Iowa, 50613. (N.S.F.)
- Newton S. Hawley, Geometry Project, Department of Mathematics, Stanford University, Stanford, California, 94305. (N.S.F.)
- Leon Henkin, Experimental Films in Mathematics (Secondary, College), Department of Mathematics, University of California, Berkeley, California, 94720. (N.S.F.)
- John R. Mayor, et al., <u>University of Maryland Mathematic Project (UMMaP)</u> (Grades 7 and 8), College Park, Maryland. (Carnegie and N.S.F.)
- David Page, Arithmetic Project (formerly University of Illinois Arithmetic Project), (Elementary), Educational Services Incorporated, 108 Water Street, Watertown, Massachusetts, 02172. (Carnegie and N.S.F.)
- Robert A. Rosenbaum, <u>High School Course in Modern Co-ordinate Geometry</u>, Department of Mathematics, Wesleyan University, Middletown, Conneticut, 06457. (N.S.F.)
- Paul C. Rosenbloom, Minnesota Mathematics and Science Teaching Project (MINNEMAST) (all grades), University of Minnesota, Minnesota, Minnesota, (Various sponsors)



Harry D. Ruderman, In-Service Films in Mathematics for Elementary School Teachers, Department of Mathematics, Hunter College High School, 930 Lexington Avenue, New York, New York, 10021. (N.S.F.)

W. Warwick Sawyer, Introducing Mathematics 1: Vision in Elementary Mathematics, Penguin Books, Baltimore, Maryland, 1964.

Joseph Struthers, et al., <u>Mathematics for the Elementary School</u>, an In-Service Course for Elementary School Teachers (Elementary), Colorado Department of Education, Office of Instructional Services, Division of Elementary and Secondary Education, Denver, Colorado, 80203.(Colorado Department of Education.)

Patrick Suppes, Experimental Teaching of Mathematics in the Elementary School, Institute for Mathematical Studies in the Social Sciences, Stanford University, Stanford, California, 94305. (N.S.F.)

Patrick Suppes, Computer Based Teaching Machines, Institute of Mathematical Studies in the Social Sciences, Stanford University, Stanford, California. (Carnegie Foundation, and International Business Machines, Inc.)

Henry Van Engen, <u>Patterns in Arithmetic</u>, Departments of Education and Mathematics, University of Wisconsin, Madison, Wisconsin, 53706. (N.S.F.)

Robert W. Wirtz, Morton Botel, W. Warwick Sawyer, <u>Mathematics Workshop</u>, Encyclopedia Brittannica Films, 425 North Michigan Ave., Chicago 11, Illinois

United States Projects -- Nursery School

Joy C. Levy, <u>Number Work for Nursery School</u>, 102 Russell Road, Princeton, New Jersey.

United States Projects -- Logic

The topic of mathematical logic is new to the pre-college curriculum, and has been approached from many quite different angles by various projects, including these:

Laymen Allen, The ALL Project (Accelerated Learning of Logic) (grade 6), Yale University, New Haven, Conneticut. (N.S.F.)

Robert B. Davis, The Madison Project (grades 5-12) Webster College, Webster Groves, Missouri 63119. (N.S.F., U.S. Office)



Suppos and Hill, First Course in Mathematical Logic (grades 6-12), Blaisdell Publishing Co., New York 1964. (N.S.F.)

John G. Kemeny, Laurie J. Snell, and Gerald L. Thompson, <u>Introduction to Finite</u> Mathematics (Secondary), Prentice-Hall, Inc., 1957

United States Projects -- Matrices

E. G. Begle, The School Mathematics Study Group (SMSG) (grade 12), Stanford University, Stanford, California.

Robert B. Davis, The Madison Project (grades 5-12), Webster College, Webster Groves, Missouri, 33119. (N.S.F. and U. S. Office)

John G. Kemeny, Laurie J. Snell and Gerald L. Thompson, <u>Introduction to Finite</u> Mathematics (Secondary), Prentice-Hall, Inc., 1957.

United States Project -- Statistics & Probability

Introductory Probability and Statistical Inference: An Experimental Course, prepared for the Commission on Mathematics, College Entrance Examination Board, 425 West 117 Street, New York 27, New York, 1959.(Carnegie Corporation)

Frederick Mosteller, Robert Rourke, George B. Thomas, Jr. Probability: A First Course, Addison-Wesley Publishing Co., Reading, Massachusetts 1961.(Continental Classroom)

United States Projects -- Calculus in Grade 12

Advanced Placement Program, College Entrance Examination Board, 425 West 117 Street, New York 27, New York.

Albert Blank, Chairman, The School Mathematics Study Group, (SMSG), Calculus for Grade 12., SMSG School of Education, Stanford University, Stanford, California, 94305.



United States Project -- Mathematics Combined with Science

Many Science projects are developing significant portions of mathematics in conjunction with their work in science. This "combination" is new, in that it goes well beyond what was common in the past. Science projects will not be listed here -- please refer to the chapter on science.

From the other side, several mathematics projects are attempting to include portions of science perhaps especially these: MINNEMAST, SMSG, and The Madison Project.

Relation to College Education

In planning pre-college mathematics programs, it is helpful (and possibly essential) to know about the very great changes in college mathematics that are beginning to take place. These effect the high school graduate as he prepares for college, and they effect the education of teachers, both pre-service and in-service.

R. C. Buck and John A. Nohel, Experimental Curriculum in Engineering Mathematics, Department of Mathematics, University of Wisconsin, Madison, Wisconsin, 53706·(N.S.F.)

Ralph Crouch and George Baldwin, <u>Mathematics Course for Prospective Elementary School Teachers</u>. Address: Ralph Crouch, Department of Mathematics, New Mexico State University, Box 396, University Park, New Mexico, 88076. George Baldwin, Department of Mathematics, Eastern New Mexico University, Portales, New Mexico, 83130. (N.S.F.)

Robert B. Davis, Undergraduate College Education in Relation to the "New Curriculum" in Mathematics, Current Issues in Higher Education, 1964, pp. 153-155.

Ainsley Fi. Diamond, <u>Undergraduate Course in Mathematical Logic</u>, Department of Mathematics, Stevens Institute of Technology, Hoboken, New Jersey, 07030. (N.S.F.)

William L. Duren, Jr., Committee on the Undergraduate Program in Mathematics (CUPM), 221 A Thornton Hall, University of Virginia, Charlottesville, Virginia, 22903. (N.S.F.)



- John G. Kemeny, Experimental Undergraduate Instruction in Computing, Department of Mathematics, Dartmouth College, Hanover, New Hampshire, 03755. (N.S.F.)
- J. Maurice Kingston, Course in Mathematics for Prospective Junior High School Teachers, Department of Mathematics, University of Washington, Seattle, Washington, 98105. (N.S.F.)
- Howard Levi, <u>Power Series and the Elementary Functions</u> (Secondary, College), Department of Mathematics, Columbia University, New York, New York, 10027. (Present address: Department of Mathematics, Hunter College, 695 Park Avenue, New York, New York, 10021. (N.S.F.)
- H. M. MaciNeille, Films and Other Teaching Materials for College Mathematics, Department of Mathematics, Case Institute of Technology, Cleveland, Ohio, 44106. (N.S.F.)
- Kenneth O. May, Experimental Prograduate Program in Mathematics, Department of Mathematics, Carleton College, Northfield, Minnesota, 55057. (N.S.F.)
- John R. Mayor and Helen L. Garstens, Mathematics Courses for Prospective Elementary School Teachers, Department of Mathematics, University of Maryland, College Park, Maryland, 20742. (N.S.F.)
- G. Baley Price, Mathematics Course for Students of the Biological, Management, and Social Sciences (Secondary, College), Department of Mathematics, The University of Kansas, Lawrence, Kansas, 66045. (N.S.F.)
- Seymour Schuster, Geometry Course for Prospective High School Mathematics
 Teachers, Minnesota School Mathematics and Science Center, University of Minnesota,
 Minnesota, 55455. (N.S.F.)
- Robert J. Walker, Experimental Teaching Program in Algebra, Department of Mathematics, Cornell University, Ithaca, New York, 14850. (N.S.F.)
- G. S. Young, New Undergraduate Courses in Mathematics, Department of Mathematics, Tulane University, New Orleans, Louisiana, 70118. (N.S.F.)

Psychological Aspects

Lee Cronbach, et al., Learning by Discovery. (Conference held in New York City, January 28 and 29, 1965. Proceedings to be published shortly.) Stanford University, Stanford, California. (U. S. Office of Education)



Jean Piaget, et. al., Conferences on Science Curriculum Planning, address: Verne N. Rockcastle, Science Education Division, Cornell University, Ithaca, New York, 14850.(N.S.F.)

Robert R. Sears and Jerome Bruner, <u>Learning about Learning</u>, Stanford University, Stanford, California, 94305. (N.S.F.)

Historical Background

As the following two references show, the point of view and goals are not really "new". What is new is the amount of money being committed, and the determination to make an all-out effort toward achieving these goals. The obstacles remain as difficult as ever.

Mary Everest Boole, The Preparation of the Child for Science, Oxford (1904).

National Committee on Mathematical Requirements of the Mathematical Association of America, The Reorganization of Mathematics in Secondary Education (1923). The Mathematical Association of America, SUNY at Buffalo, Buffalo, New York, 14214.

International and Foreign Projects

- Z. P. Dienes, Adelaide Mathematics Project (Elementary grades), University of Adelaide, Adelaide, Australia.
- W. T. Martin, The Entebbe Mathematics Workshop (Grades 1-12, plus teacher education materials), Educational Services Inc. (A High-quality modern mathematics program for the English-speaking nations of Africa, based upon United States "new mathematics" efforts; also useful in the United States.) (U.S. A.I.D.)
- George H. Matthews, Mathematics Teaching Project (all grades), Nuffield Foundation, St. Dunstan's College, London, England. (Nuffield Foundation)
- Alfred L. Putnam and Izaak Wirzup, Survey of Recent East European Literature on School and College Mathematics, Department of Mathematics, The University of Chicago, Chicago, Illinois, 60637. (N.S.F.)



Leonard Sealey, Leicestershire Mathematics Project (Grades 1-6), Education Department, Grey Friars, Leicester, Great Britain

Commericial Publications

Because mathematics has been involved in new curriculum work for at least ten years of intense activity, significant portions of this activity have reached commercial publication or influenced commercial publication. It is important to recognize this. The work of "projects" is not limited to the direct output of projects.

Not all commercial publications that claim to be "modern" are truly entitled to make such a claim, but some are. Decisions on commercial texts, pamphlots, and supplementary materials are best left entirely to local personnel, with perhaps the remark that Addison-Vesley and S. C. Heath have been particularly venturesome in bringing out materials of high quality and genuinely modern content.

